Solution by the PROPOSER.

The probable error for a, $=r\sqrt{a}$; for b, $r\sqrt{b}$; for c, $r\sqrt{c}$.

$\therefore$ Error of volume in length$=bcr\sqrt{a}$.

Error of volume in width$=acr\sqrt{b}$.

Error of volume in thickness$=abr\sqrt{c}$.

The probable error of volume=square root of the sum of the squares of these three errors.

$\therefore$ Probable error$=\sqrt{[(ab^2c^2+a^2bc^2+a^2b^2c)r^2]}=r\sqrt{[abc(ab+ac+bc)]}$.

MISCELLANEOUS.

124. Proposed by J. W. YOUNG, Graduate Student, Cornell University, Ithaca, N. Y.

Prove that the general value of θ, which satisfies the equation

$$(\cos\theta+i\sin\theta)(\cos2\theta+i\sin2\theta)\ldots\text{to } n \text{ factors}=1 \text{ is } \frac{4m\pi}{n(n+1)};$$

where m is any integer ($i=\sqrt{-1}$).

Solution by G. W. GREENWOOD, A. M., McKendree College, Lebanon, Ill.; LON C. WALKER, A. M., Leland Stanford Jr. University, Cal., and J. SCHEFFER, A. M., Hagerstown, Md.

$$1=(\cos\theta+i\sin\theta)(\cos2\theta+i\sin2\theta)\ldots(\cos n\theta+i\sin n\theta)=(\cos\theta+i\sin\theta)^{1+2+\ldots+n}$$

$$=(\cos\theta+i\sin\theta)^{\frac{1}{2}[n(n+1)]}=\cos\frac{n(n+1)\theta}{2}+i\sin\frac{n(n+1)\theta}{2}.$$

$$\therefore \frac{n(n+1)\theta}{2}=2m\pi, \text{ where } m \text{ is any integer; } i.\ e.\ \theta=\frac{4m\pi}{n(n+1)}.$$

Also solved by *G. B. M. ZERR.*

125. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Assume $m=nt+\varepsilon-\omega$, thus giving $v=m+e\sin v$ as the relation connecting the mean and eccentric anomalies, then express $x=a\cos v$, $y=b\sin v$, and $r=a(1-e\cos v)$ by a Fourier series in terms of m.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $y_1=z+x\varphi((y)$, we get by Lagrange's Theorem,

$$f(y_1)=f(z)+x\varphi(z)f'(z)+\frac{x^2}{1.2}\frac{d}{dz}\{[\varphi(z)]^2f'(z)\}+\frac{x^3}{1.2.3}\left(\frac{d}{dz}\right)^2\{[\varphi(z)]^3f'(z)\}+$$
etc., etc.

From $v=m+e\sin v$, $y_1=v$, $z=m$, $x=e$, $\varphi(y)=\sin v$.

Now $f(v)=v$ and $f'(v)=1$.

$$\therefore\ y_1=z+x\sin z+\frac{x^2}{1.2}\frac{d}{dz}(\sin^2 z)+\frac{x^3}{1.2.3}\left(\frac{d}{dz}\right)^2(\sin^3 z)+\text{ etc.}$$

$$\therefore\ y_1=z+x\sin z+\frac{x^2}{1.2}\frac{d}{dz}\left(\frac{1-\cos 2z}{2}\right)+\frac{x^3}{1.2.3}\left(\frac{d}{dz}\right)^2\left(\frac{3\sin z-\sin 3z}{4}\right)$$

$$+\frac{x^4}{1.2.3.4}\left(\frac{d}{dz}\right)^3\left(\frac{3-4\cos 2z+\cos 4z}{8}\right)+\text{ etc.}$$

$$=z+x\sin z+\tfrac{1}{2}x^2\sin 2z+\tfrac{1}{8}x^3(3\sin 3z-\sin z).$$

$$\therefore\ v=m+e\sin m+\tfrac{1}{2}e^2\sin 2m+\tfrac{1}{8}e^3(3\sin 3m-\sin m)$$

$$+\tfrac{1}{6}e^4(2\sin 4m-\sin 2m)+\text{ etc. }=m+e\sin v.$$

$$\therefore\ \sin v=\sin m+\tfrac{1}{2}e\sin 2m+\tfrac{1}{8}e^2(3\sin 3m-\sin m)+\tfrac{1}{6}e^3(2\sin 4m-\sin 2m)+\text{etc.}$$

To develop $(1-e\cos v)$ in terms of m: Let $f(y_1)=1-e\cos y_1$, $f'(y_1)=e\sin y_1$.

$$\therefore\ 1-e\cos y_1=(1-e\cos z)+x\sin z(e\sin z)\ +\frac{x^2}{1.2}\frac{d}{dz}(\sin^2 z.e\sin z)+\text{ etc.}$$

Performing the operations as before we get after substituting v for y_1, m for z, e for x,

$$1-e\cos v=1-e\cos m+\tfrac{1}{2}e^2(1-\cos 2m)+\tfrac{1}{8}e^3(3\cos m-3\cos 3m)$$

$$+\tfrac{1}{3}e^4(\cos 4m-\cos 2m)+\text{ etc.}$$

$$\therefore\ \cos v=\cos m+\tfrac{1}{2}e(\cos 2m-1)+(3e^2/8)(\cos 3m-\cos m)$$

$$+\tfrac{1}{3}e^3(\cos 2m-\cos 4m)+\text{ etc.}$$

$$\therefore\ \sin v=A\sin m+B\sin 2m+C\sin 3m+D\sin 4m+\ldots.$$
$$\cos v=-\tfrac{1}{2}e+A_1\cos m+B_1\cos 2m+C_1\cos 3m+D_1\cos 4m+\ldots.$$

where A, B, C, D, ...A_1, B_1, C_1, D_1, ... are each a series in powers of e.

$$\therefore\ x=a\cos v=-\tfrac{1}{2}ae+aA_1\cos m+aB_1\cos 2m+aC_1\cos 3m+aD_1\cos 4m+\ldots.$$
$$y=b\sin v=bA\sin m+bB\sin 2m+bC\sin 3m+bD\sin 4m+\ldots.$$
$$r=a(1-e\cos v)=a[1+\tfrac{1}{2}e^2-eA_1\cos m-eB_1\cos 2m-eC_1\cos 3m$$
$$-eD_1\cos 4m+\ldots.].$$

126. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

The declination of a certain fixed star if 12° 40′. Its altitude was observed one day to be 16° 40′. Three hours and twenty-four minutes later it was found to be 40° 20′. Find the latitude of the place of observation.